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# QCD-Gravity correspondence

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BNL



# Outline

1. Scale invariance and its breaking
2. Shear and bulk viscosities
3. Gravity and black holes
4. AdS/CFT correspondence:

N=4 SUSY Yang-Mills  Gravity

Example: shear viscosity

5. QCD  Scale anomaly  Gravity

Example: bulk viscosity

# 1. Scale invariance in field theory

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# Example: Maxwell electrodynamics

The Lagrangian

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

The energy-momentum tensor:

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are needed to see this picture.

Since QuickTime™ and a  
TIFF (Uncompressed) decompressor  
are needed to see this picture. in four dimensions, the trace of the energy-momentum tensor vanishes -

Classical electrodynamics is scale invariant  
(in the absence of external sources)

Yang-Mills theory is also scale invariant classically,  
but not quantum-mechanically:  $g^2(\mathbf{r})$

# The scale anomaly of QCD

$$\mathcal{L} = -\frac{1}{4}G_{\mu\nu}^a G_{\mu\nu}^a + \sum_f \bar{q}_f^a (i\gamma_\mu D_\mu - m_f) q_f^a;$$

Classical scale invariance is broken by quantum effects:

scale anomaly

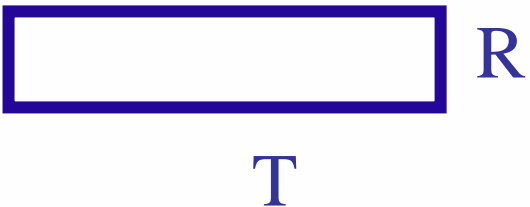
$$\theta_\mu^\mu = \frac{\beta(g)}{2g} G^{\alpha\beta a} G_{\alpha\beta}^a + \sum_q m_q \bar{q}q$$

trace of the energy-momentum tensor

“beta-function”; describes the dependence of coupling on momentum

Hadrons get masses  $\longleftrightarrow$  coupling runs with the distance

# Scale invariance and confinement

Consider a rectangular Wilson loop: 

$$W(C) = \exp \left( ig \int_C A_\mu dx^\mu \right)$$

It is related to the potential  $V(R)$  acting between the charges  $Q$  and  $\bar{Q}$ :

$$W(C) \rightarrow \exp(-TV(R))$$

Scale transformation:  $T \rightarrow \lambda T$ ;  $R \rightarrow \lambda R$ ;

the only solution: Coulomb potential

$$V(R) \sim \frac{1}{R}$$

# Scale invariance and confinement

$$W(C) \rightarrow \exp(-TV(R)) \quad \boxed{\phantom{R}} \quad R$$

T

Running coupling and especially confinement

(area law for the Wilson loop)

e.g. the potential

$$V(R) = -\frac{4}{3} \frac{\alpha_s(R)}{R} + \sigma R$$

explicitly break scale invariance

## 2. Shear and bulk viscosities: the definitions

The energy-momentum tensor:

$$\theta_{ij} = P_{eq}(\epsilon)\delta_{ij} - \eta \left( \partial_i u_j + \partial_j u_i - \frac{2}{3}\delta_{ij}\partial_k u_k \right) - \zeta \delta_{ij} \vec{\nabla} \cdot \vec{u}$$

  
shear viscosity

  
bulk viscosity

# Kubo's formula:

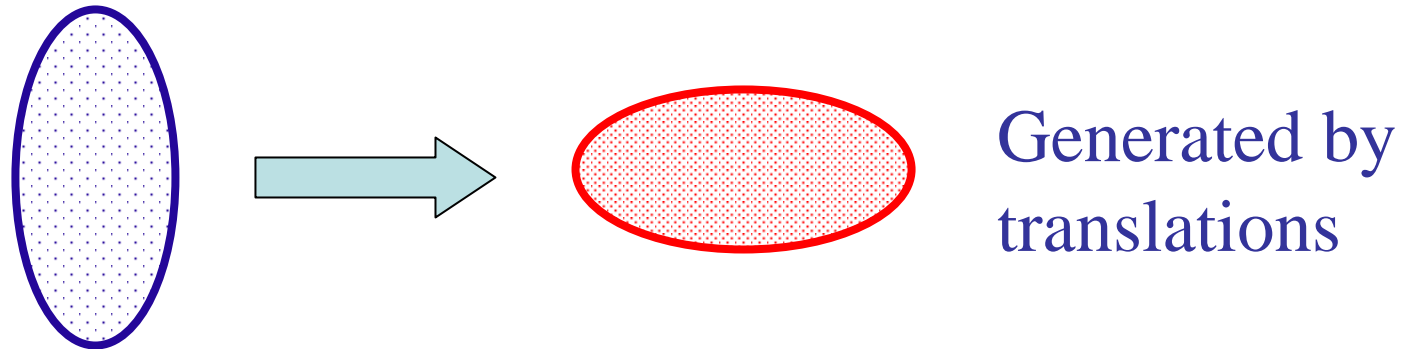
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Viscosities are defined as the static limit of  
the correlation functions, e.g. bulk viscosity:

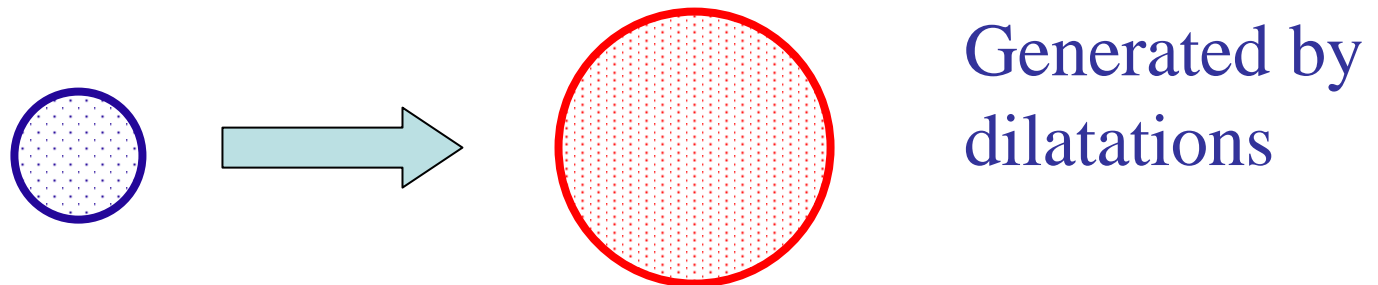
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## Physical picture:

Shear viscosity: how much entropy is produced by transformation of shape at constant volume



Bulk viscosity: how much entropy is produced by transformation of volume at constant shape



### 3. Black holes

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Rev. John Michell (1724-1793)

Letter to the Royal Society, 1783:

If the semi-diameter of a sphere of the same density as the Sun in the proportion of five hundred to one, and by supposing light to be attracted by the same force in proportion to its [mass] with other bodies,  
**all light emitted from such a body would be made to return towards it, by its own proper gravity.**

# Who was John Michell?

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Rev. John Michell (1724-1793)

**John Michell is a little short Man, of a black Complexion and fat; but having no Acquaintance with him, can say little of him...**

from a contemporary diary

# What is a black hole?

Pierre-Simon LaPlace (1749-1827)

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“What we know is not much.  
What we do not know is immense.”



"...[It] is therefore possible that the largest luminous bodies  
the universe may, through this cause, be invisible."  
-- Le Système du Monde, 1796

# What is a black hole?

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are needed to see this picture.

A. Einstein, 1915:  $G_{\mu\nu} = 8\pi T_{\mu\nu}$

Einstein tensor:

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R$$

Ricci tensor

energy-  
momentum  
tensor

K. Schwarzschild, 1916:

A solution for a static isotropic  
gravitational field

$$ds^2 = c^2 dt^2 \left(1 - \frac{2GM}{rc^2}\right) - \frac{dr^2}{1 - \frac{2GM}{rc^2}} - r^2 d^2\Omega$$

singularity at  $r = 2GM$  !

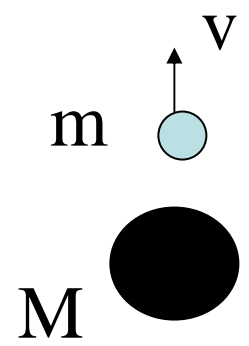
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# Schwarzschild radius

$$ds^2 = c^2 dt^2 \left(1 - \frac{2GM}{rc^2}\right) - \frac{dr^2}{1 - \frac{2GM}{rc^2}} - r^2 d^2\Omega$$

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The meaning of  $R = 2GM$  ?



Consider the following Michell-like “derivation”:

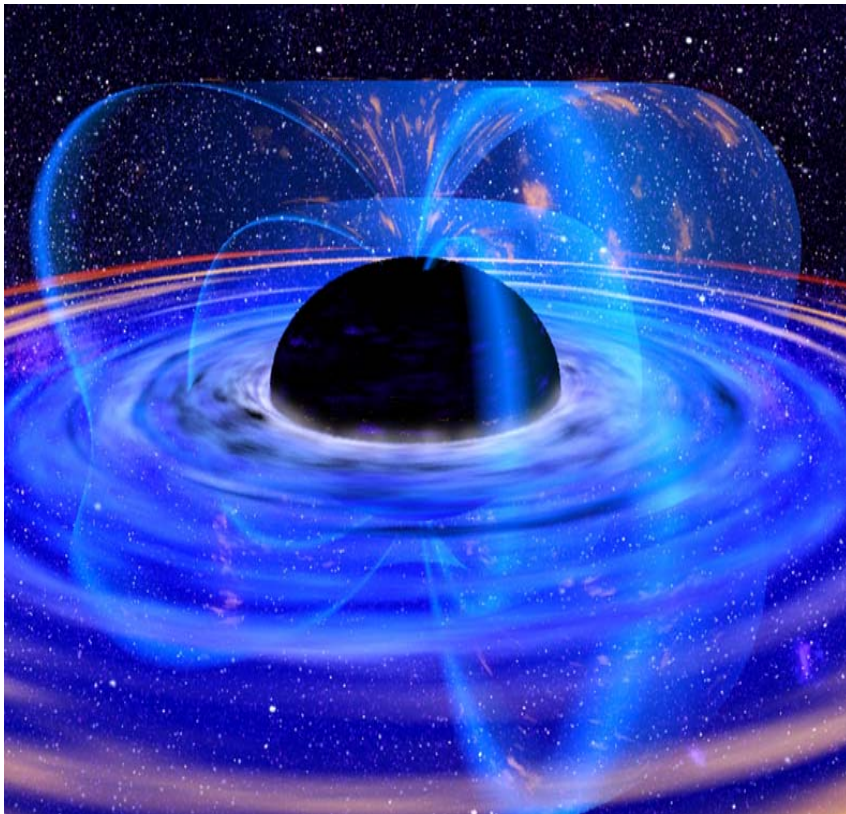
$$\frac{1}{2}mv^2 = G \frac{Mm}{R} \implies v_{escape} = \sqrt{\frac{2GM}{R}}$$

if we could put  $v = c$ , would get  $R = 2GM$   
(cannot do that!)

But: it **is** the distance at which the energy in the gravitational field is  $\sim mc^2$  ... **quantum effects?**

# Black holes radiate

S.Hawking '74



Black holes emit  
thermal radiation  
with temperature

$$T = \frac{\kappa}{2\pi}$$

acceleration of gravity  
at the surface,  $(4GM)^{-1}$

# Do we see black holes in the Universe?

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Cygnus X-1 binary system:  
super-giant star orbiting around a black hole?

# Can black holes be produced at RHIC?

$$ds^2 = c^2 dt^2 \left(1 - \frac{2GM}{rc^2}\right) - \frac{dr^2}{1 - \frac{2GM}{rc^2}} - r^2 d^2\Omega$$

RHIC:

$$M < 10^4 \text{ GeV}$$

$$R > 10^{-2} \text{ fm}$$

$$E = 200 \text{ GeV}$$

No role

for classical

or quantum

gravity:

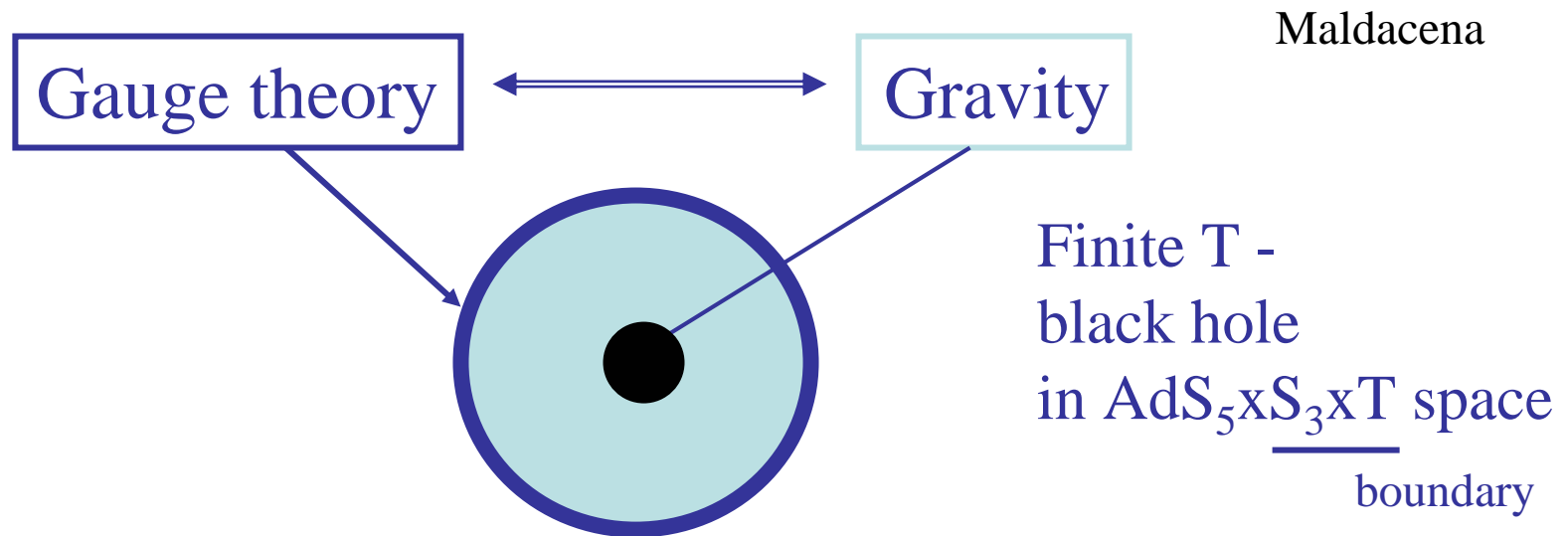
$$k_{\text{cl}} \equiv \frac{2GM}{Rc^2} < 10^{-22}$$

$$k_{\text{qu}} \equiv \frac{GE^2}{\hbar c^5} < 10^{-34}$$

W. Busza, R.L. Jaffe, J. Sandweiss,  
F. Wilczek, hep-ph/9910333

# 4. Can black holes anyway help us to understand gauge theories?

One new idea: use a mathematical correspondence between a conformal gauge theory and gravity in AdS space



Anti de Sitter space - solution of Einstein's equations with a negative cosmological constant  $\Lambda$   
(de Sitter space - solution with a positive  $\Lambda$  (inflation))

Witten; Polyakov;  
Gubser, Klebanov;  
Son, Starinets, Kovtun;  
Nastase; ...

# AdS/CFT Correspondence:

## Conformal gauge theory is dual to gravity

The chain of ideas:

1. String model pre-dates QCD as a description of hadron spectrum and hadron scattering amplitudes  
(Regge poles, Veneziano amplitude, Hagedorn spectrum,...)
2. Quantum theory of strings cannot be made consistent in (3+1) dimensions; the minimum number of dimensions is 5.  
Also: massless spin 2 state - graviton?
3. It has long been expected that QCD in the strong coupling and large  $N$  is described by string theory  
(at large  $N$ , planar diagrams dominate;  
they describe worldsheets of strings)

4. How to make this string description consistent on the quantum level?

Add a 5th dimension to QCD and let gravity live there  
BUT: what is the metric?

$$ds^2 = R^2 w^2(z) ( dx_{3+1}^2 + dz^2 )$$

what is the form of  $w(z)$ ?

5. Consider instead a conformal theory, such as maximally super-symmetric N=4 Yang-Mills; then the requirement of conformal invariance fixes the metric of the 5th dimension uniquely - it is an Anti-de Sitter space  $AdS_5$ :

invariance w.r.t.  $x \rightarrow \lambda x$  fixes  $w(z) = \frac{1}{z}$

supersymmetry -  $S_5$ , so the theory lives in  $AdS_5 \times S_5$

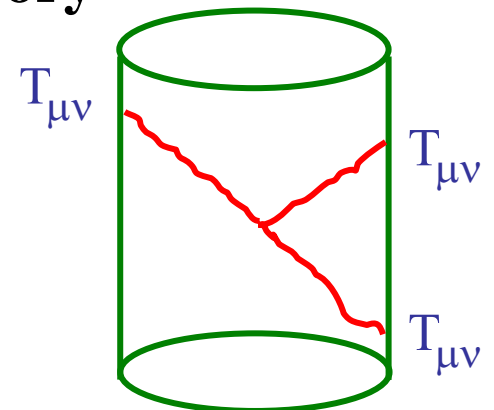
$$\boxed{N = 4 \text{ SU}(N) \text{ Yang-Mills theory}} = \boxed{\text{String theory on AdS}_5 \times S^5}$$

Radius of curvature

$$R_{S^5} = R_{AdS_5} = (g_{YM}^2 N)^{1/4} l_s$$

### Duality:

$g^2 N$  small  $\rightarrow$   $R$  small  $\rightarrow$  perturbation theory  
 $g^2 N$  large  $\rightarrow$   $R$  large  $\rightarrow$  classical gravity



# AdS/CFT and the shear viscosity bound

Kovtun, Son,  
Starinets '04

Kubo's formula relates shear viscosity to the correlation function of the energy-momentum tensor:

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are needed to see this picture.

On the other hand, the graviton absorption cross section can be expressed through the same quantity:

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where

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Combining these equations, we get

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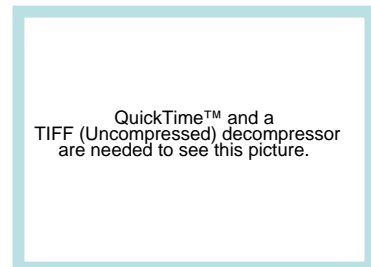
But the cross section of graviton absorption by a black hole  
is equal to the area of the horizon **a**:

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Since the entropy of a black hole is

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we find

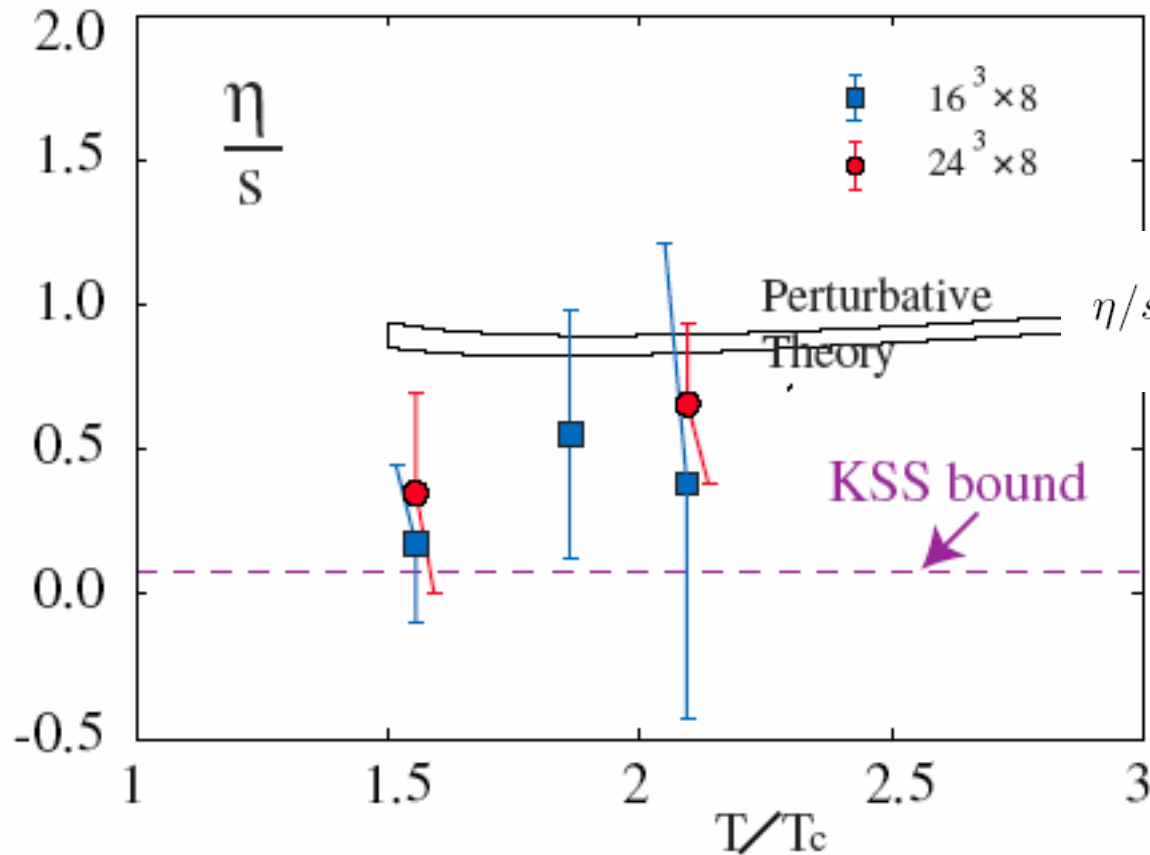


which applies in the limit of strong  
coupling, where the gravity description  
is appropriate

# Shear viscosity: lattice data

A.Nakamura and S.Sakai,  
hep-lat/0406009;

Recent work:  
H.Meyer, 0704.1801



$$\eta/s = \begin{cases} 0.134(33) & (T = 1.65T_c) \\ 0.102(56) & (T = 1.24T_c) \end{cases}$$

Perfect liquid

Limit from RHIC data?

# 5. QCD and gravity

The chain of ideas:

1. QCD is scale invariant only at the classical level; quantum effects break scale invariance (scale anomaly)
2. However, Renormalization Group implies an infinite chain of Ward identities for the correlation functions of the trace of the energy-momentum tensor
3. These Ward identities uniquely define an effective low-energy theory of QCD
4. This effective low-energy theory can be mathematically re-formulated as

**classical QCD on a curved conformal  
gravitational background**

# Ward identities: a sketch of the derivation

Consider an operator with a canonical dimension  $d$ :

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Novikov, Shifman,  
Vainshtein, Zakharov '81

The dependence of QCD Lagrangian on the coupling:

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Write down an expectation value for  $O$  as  
a functional integral and differentiate w.r.t.  $1/4g^2$ :

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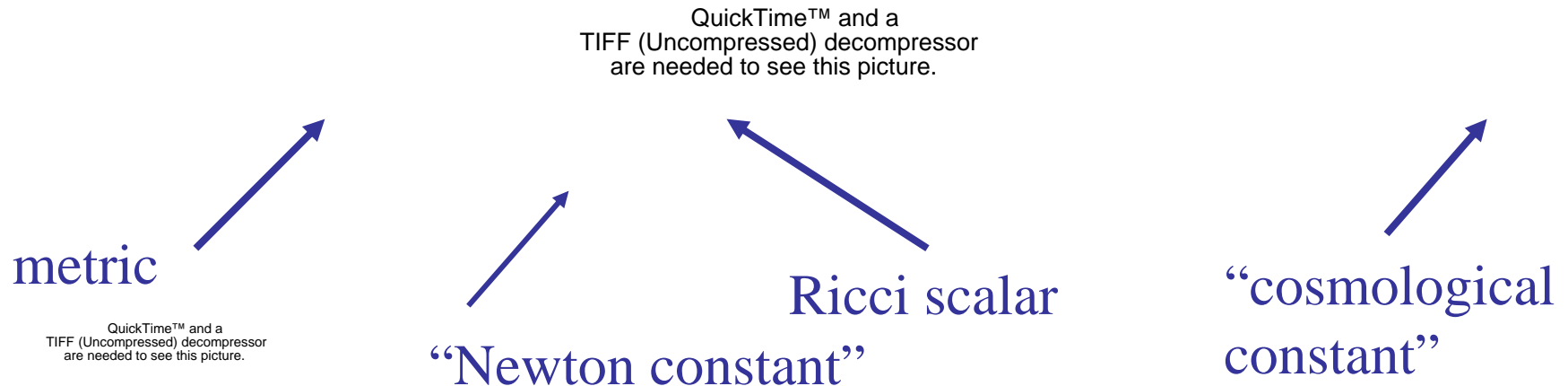
Repeat  $n$  times - get  $n$ -point correlation functions

# QCD and gravity

Elegant geometrical interpretations -  
dilaton gravity                      Migdal, Shifman '82;

classical Yang-Mills theory  
on a curved conformal gravitational background -

Einstein-Hilbert action                      DK, Levin, Tulin '04



# Confinement as an event horizon for colored particles?

Consider the metric:

P. Castorina,  
DK, H. Satz, '07

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where primes indicate first and second order derivatives with respect

to QuickTime™ and a  
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The condition

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defines the compact region of the theory, i.e.

**the counterpart of black hole**

The dependence of QCD action on  $F$  is given by

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If one-loop beta-function is used,  
the event horizon emerges at

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or at the distance

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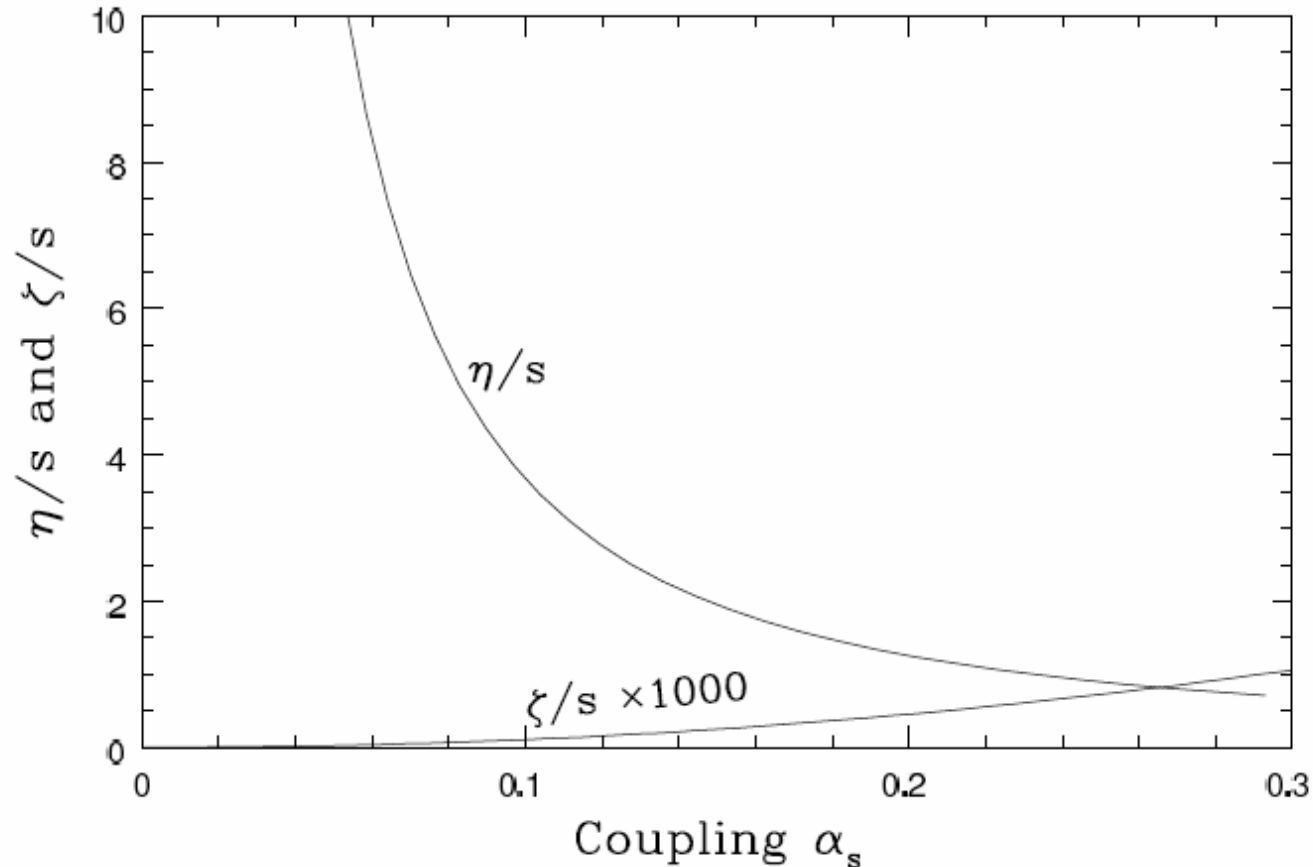
**Event horizon = Hawking temperature;**  
**The origin of thermal hadron production???**

Bulk viscosity is generated by the breaking of scale invariance; can we compute it in QCD?

Bulk viscosity is determined by the correlation function of the trace of the energy-momentum tensor:

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# Perturbation theory: bulk viscosity is negligibly small



$$\zeta \sim \frac{\alpha_s^2 T^3}{\log[1/\alpha_s]} \quad (m_0 \ll \alpha_s T)$$

P.Arnold, C.Dogan,  
G.Moore, hep-ph/0608012

In perturbation theory, shear viscosity is “large”:

$$\frac{\eta}{s} \sim \frac{1}{\alpha_s^2}$$

and bulk viscosity is “small”:

$$\frac{\zeta}{s} \sim \alpha_s^2$$

At strong coupling,  $\eta$  is apparently small;

can  $\zeta$  get large?

# An exact sum rule for bulk viscosity

Basing on LET's and Kubo's formula, we derive an exact sum rule for the spectral density:

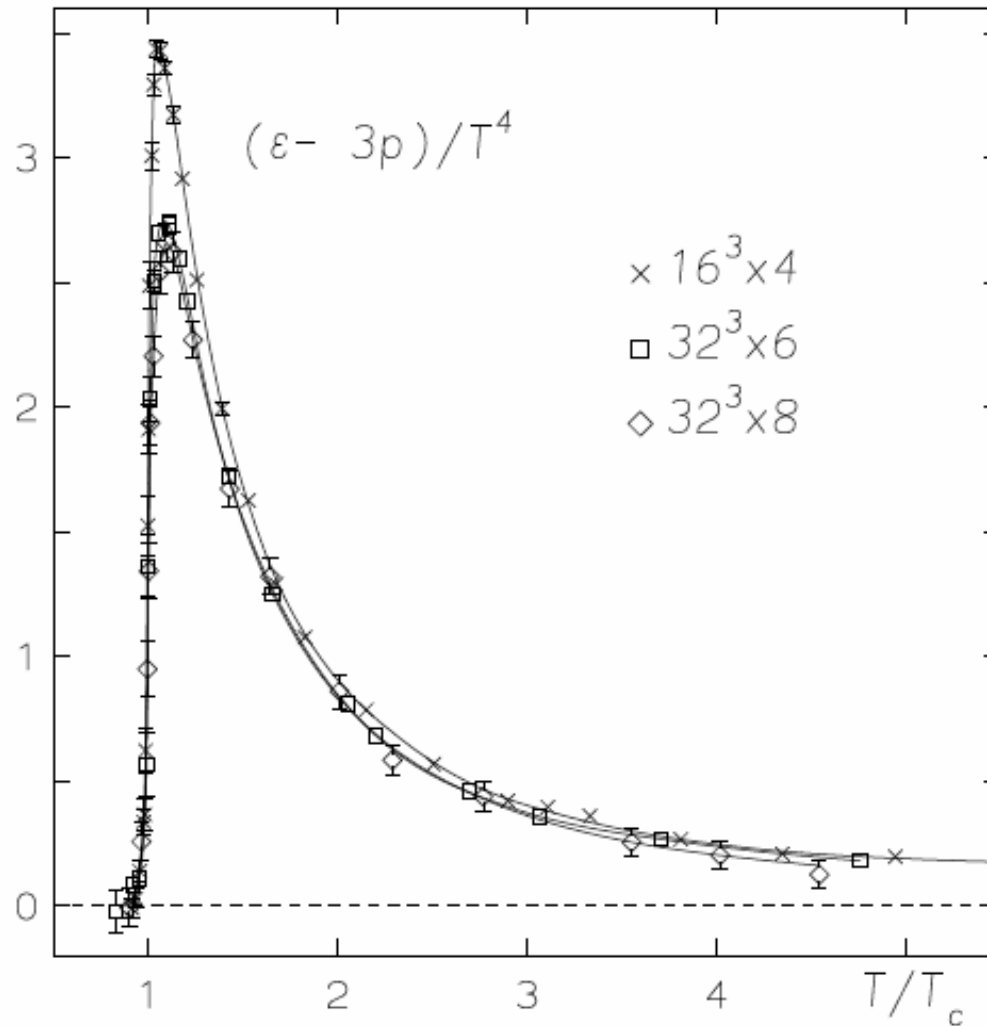
$$2 \int_0^\infty \frac{\rho(u, \vec{0})}{u} du = - \left( 4 - T \frac{\partial}{\partial T} \right) \langle \theta \rangle_T = T^5 \frac{\partial}{\partial T} \frac{(\mathcal{E} - 3P)_{\text{LAT}}}{T^4} + 16|\epsilon_v|$$

Using ansatz

we get

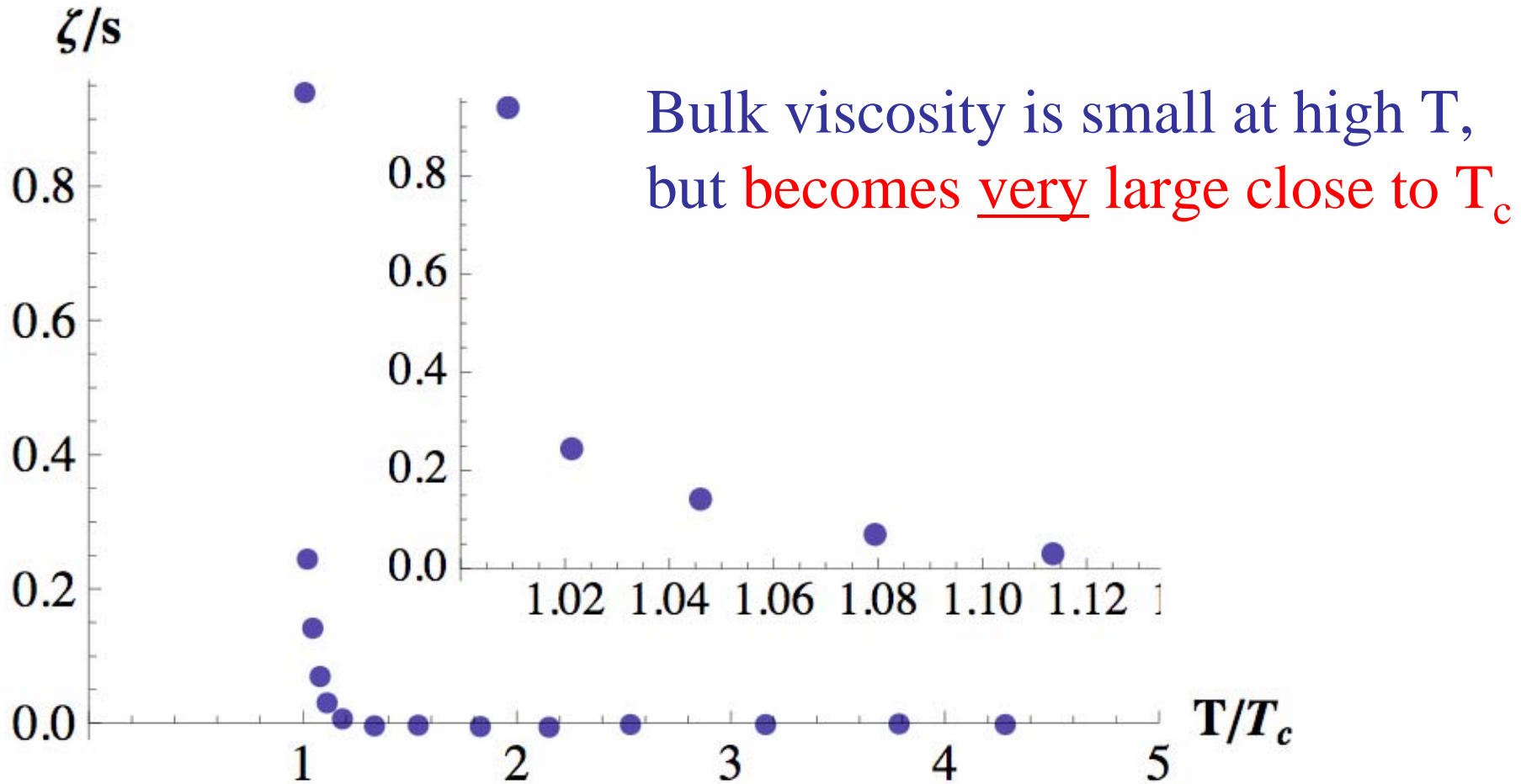
$$\frac{\rho(\omega, \vec{0})}{\omega} = \frac{9\zeta}{\pi} \frac{\omega_0^2}{\omega_0^2 + \omega^2} \quad \zeta = \frac{1}{9\omega_0} \left\{ T^5 \frac{\partial}{\partial T} \frac{(\mathcal{E} - 3P)_{\text{LAT}}}{T^4} + 16|\epsilon_v| \right\}$$

SU(3),  
pure gauge



Use the lattice data from G.Boyd, J.Engels, F.Karsch, E.Laermann,  
C.Legeland, M.Lutgeimer, B.Petersson, hep-lat/9602007

# The result



# Condensed matter analogies?

Example:  $^3\text{He}$  near the critical point

at  $(T-T_c)/T_c = 10^{-4}$  on the critical isochore,

shear viscosity is  $\eta = 17 \cdot 10^{-6}$  Poise

whereas bulk viscosity is  $\zeta = 50$  Poise

**The ratio  $\zeta/\eta$  is in excess of a million**

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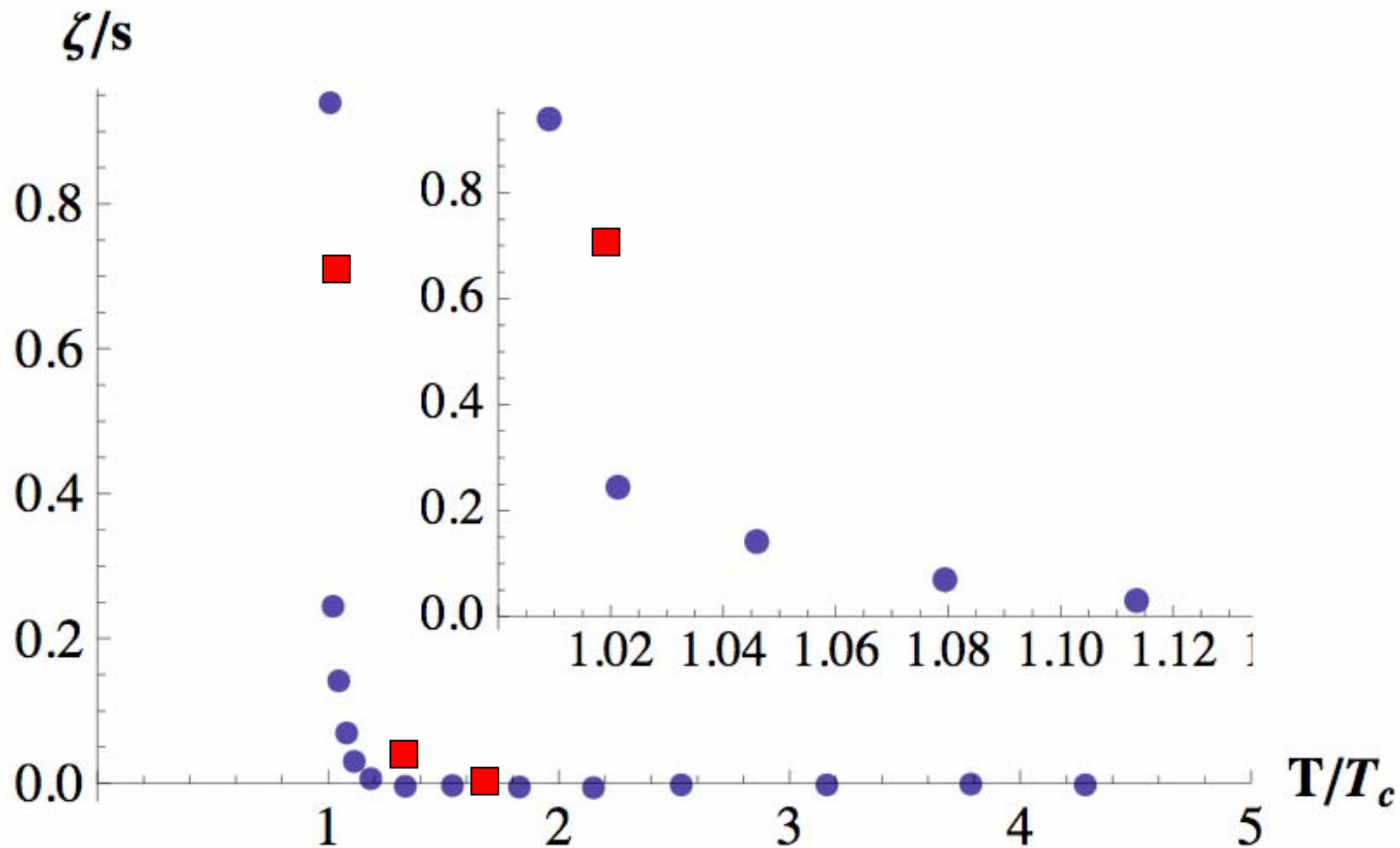
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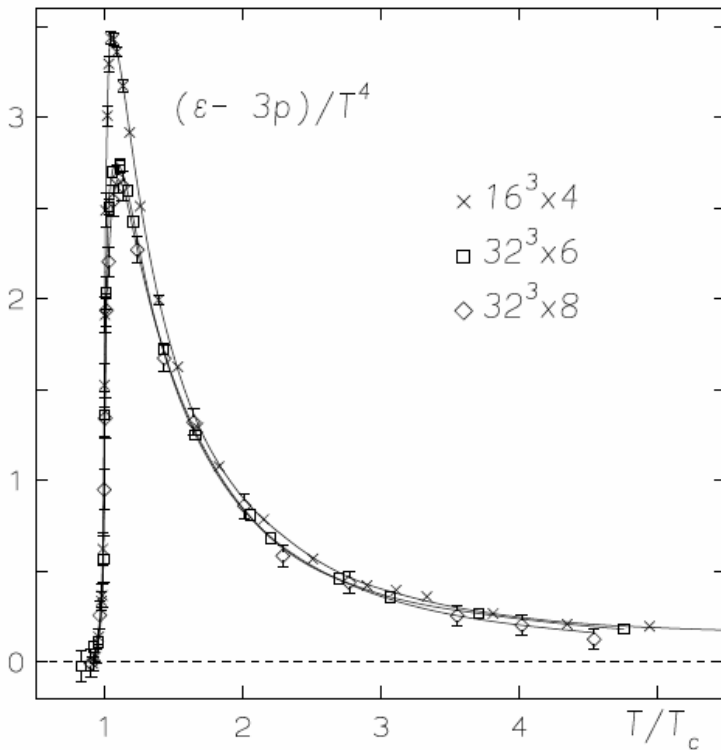
● Kharzeev-Tuchin

■ Meyer

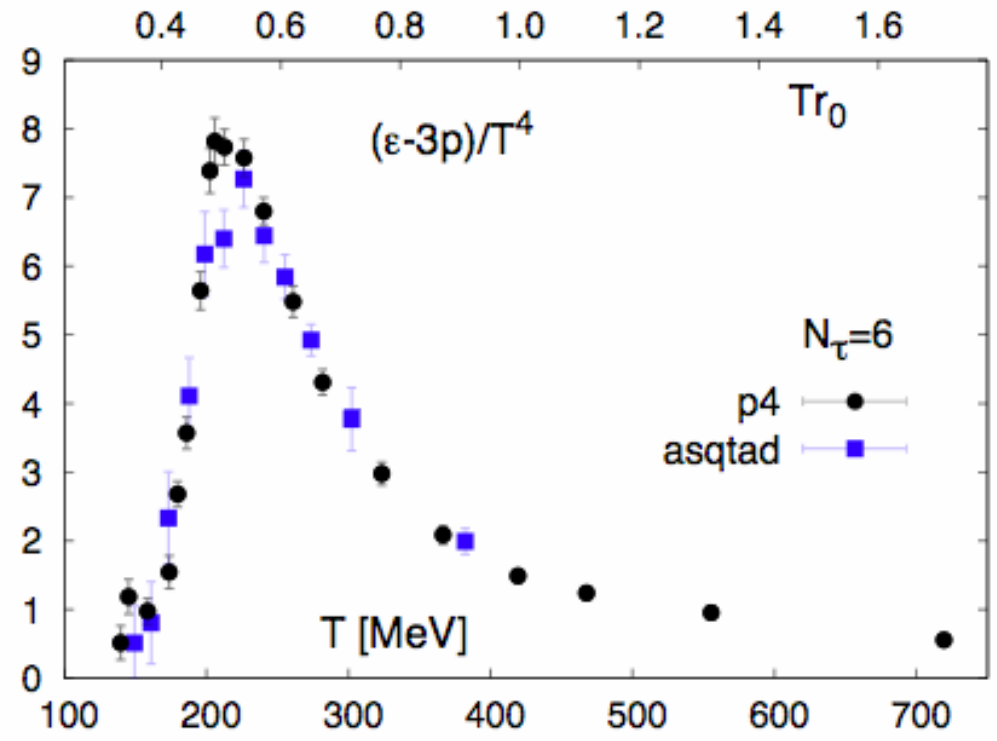


# Bulk viscosity in full QCD

Qualitatively similar results: F.Karsch, DK, K.Tuchin



SU(3), pure gauge



QCD, 2+1 quark flavors (pion mass 220 MeV)

BNL-Columbia-RBRC-Bielefeld

arXiv:0710.0354

# Bulk viscosity in full QCD

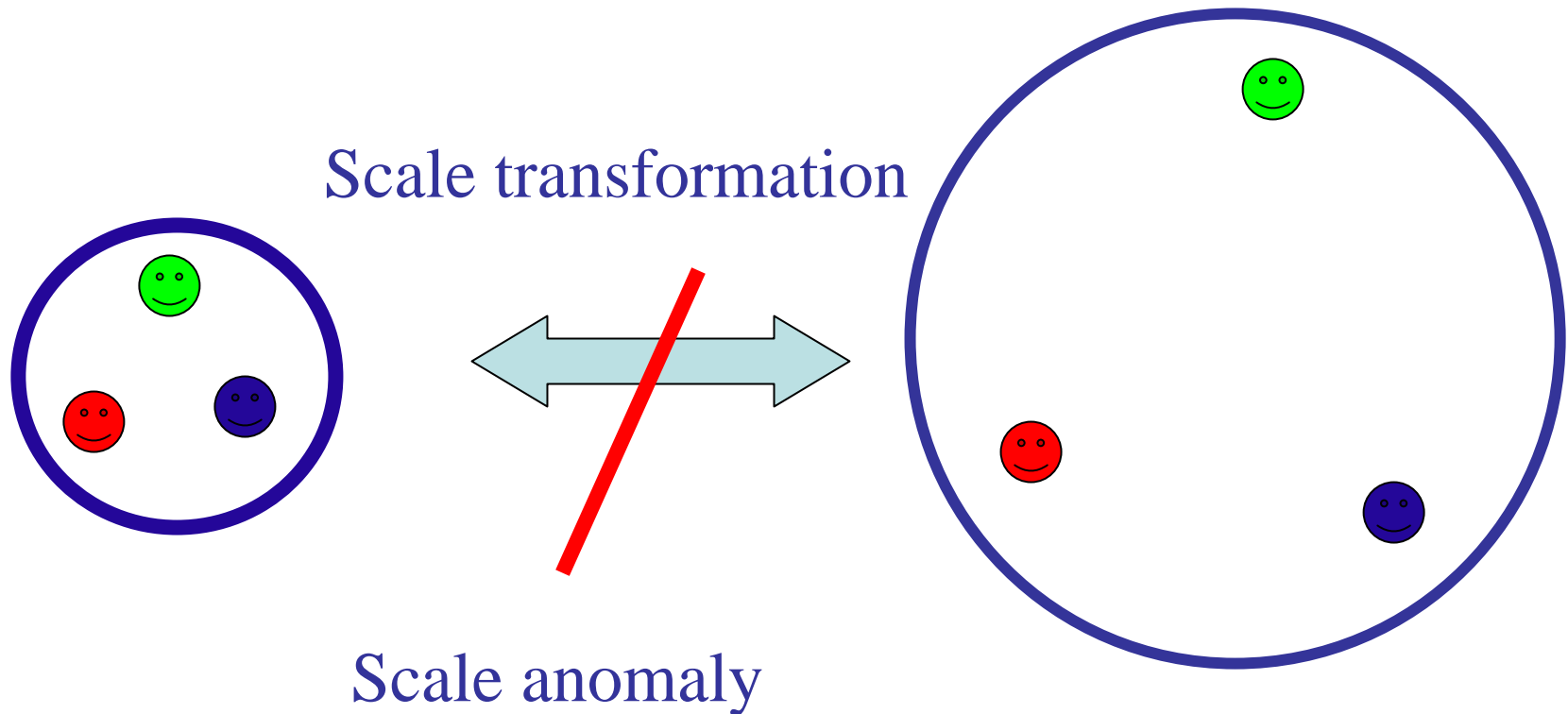
Qualitatively similar results:

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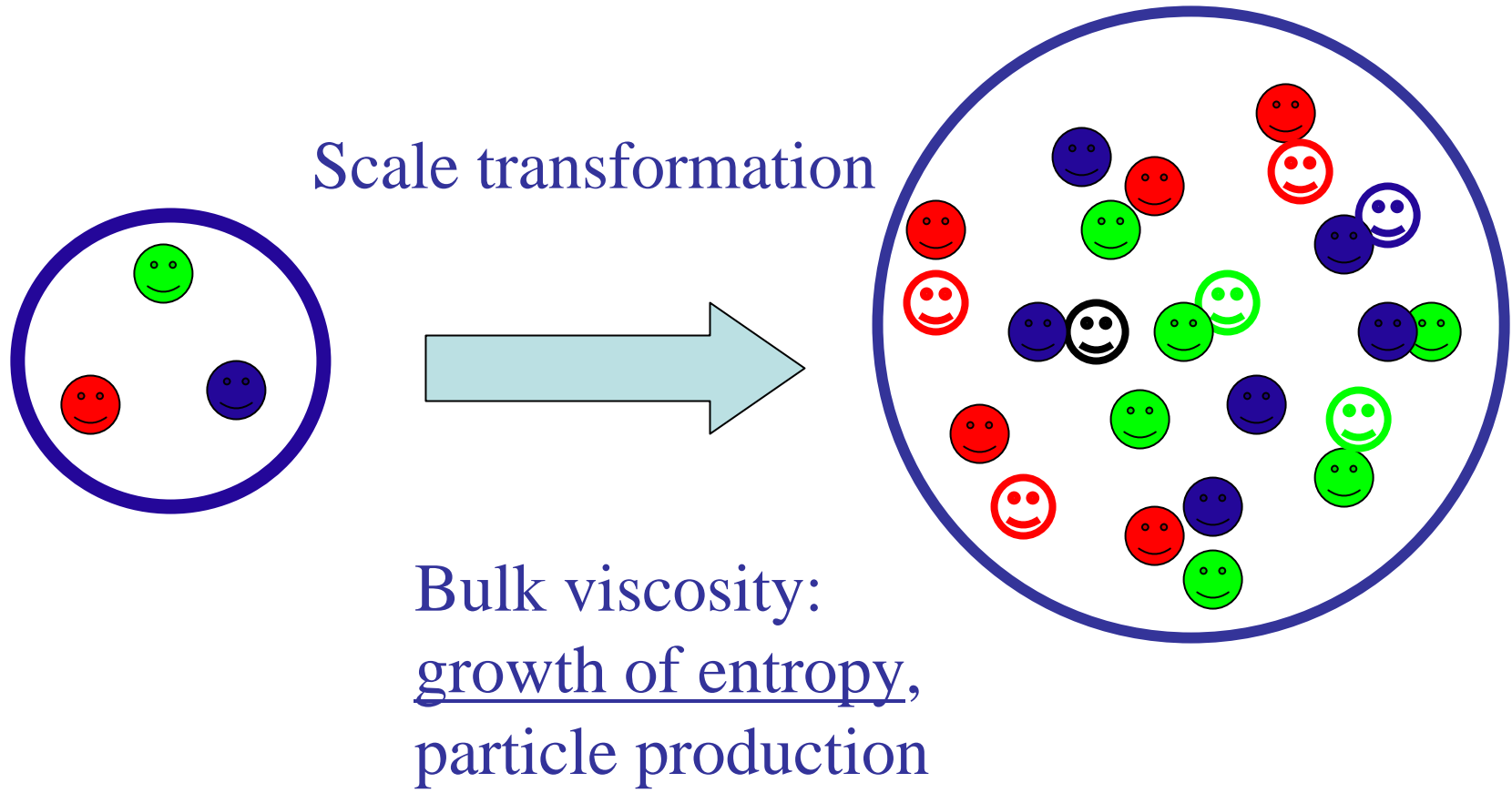
+ Near the chiral critical point: divergence of bulk viscosity

# Bulk viscosity and the mechanism of hadronization



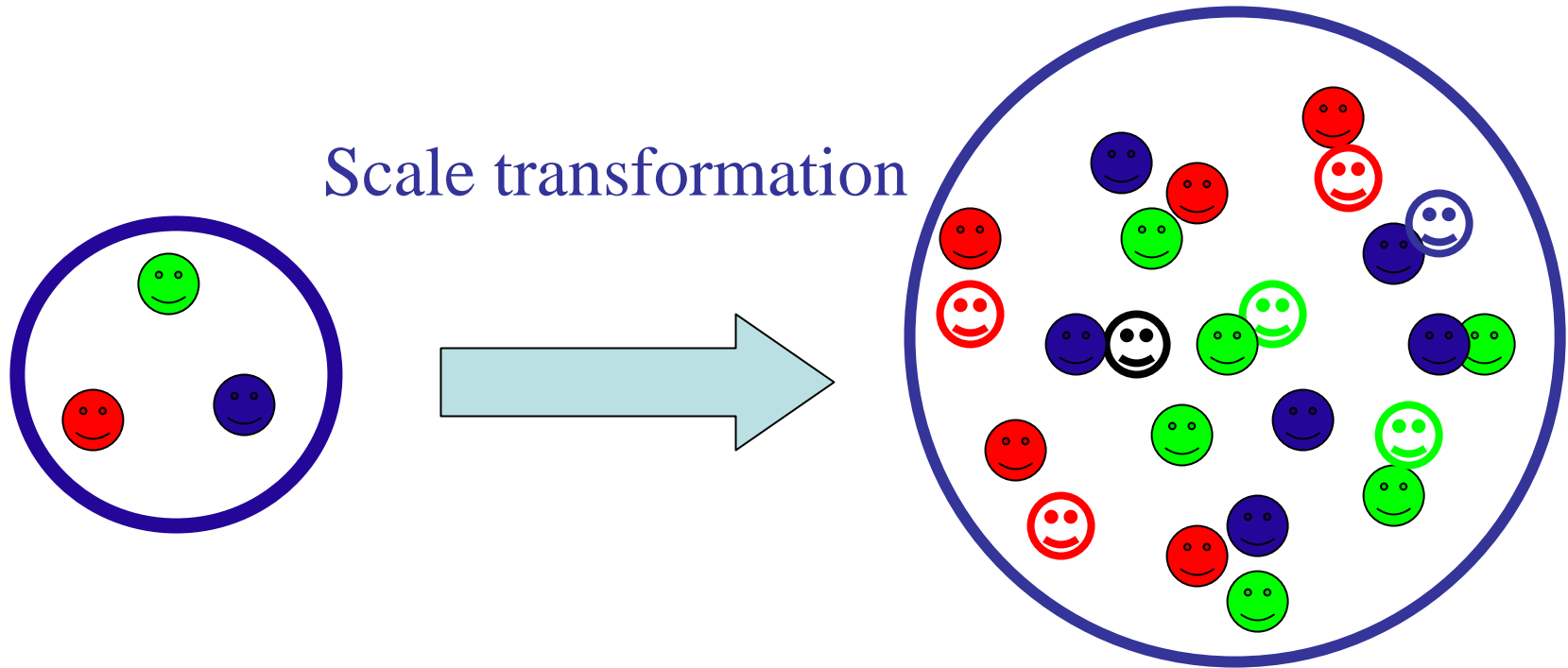
What is the meaning of the bulk viscosity growth?

# Bulk viscosity and the mechanism of hadronization



Bulk viscosity growth = soft statistical hadronization (?)

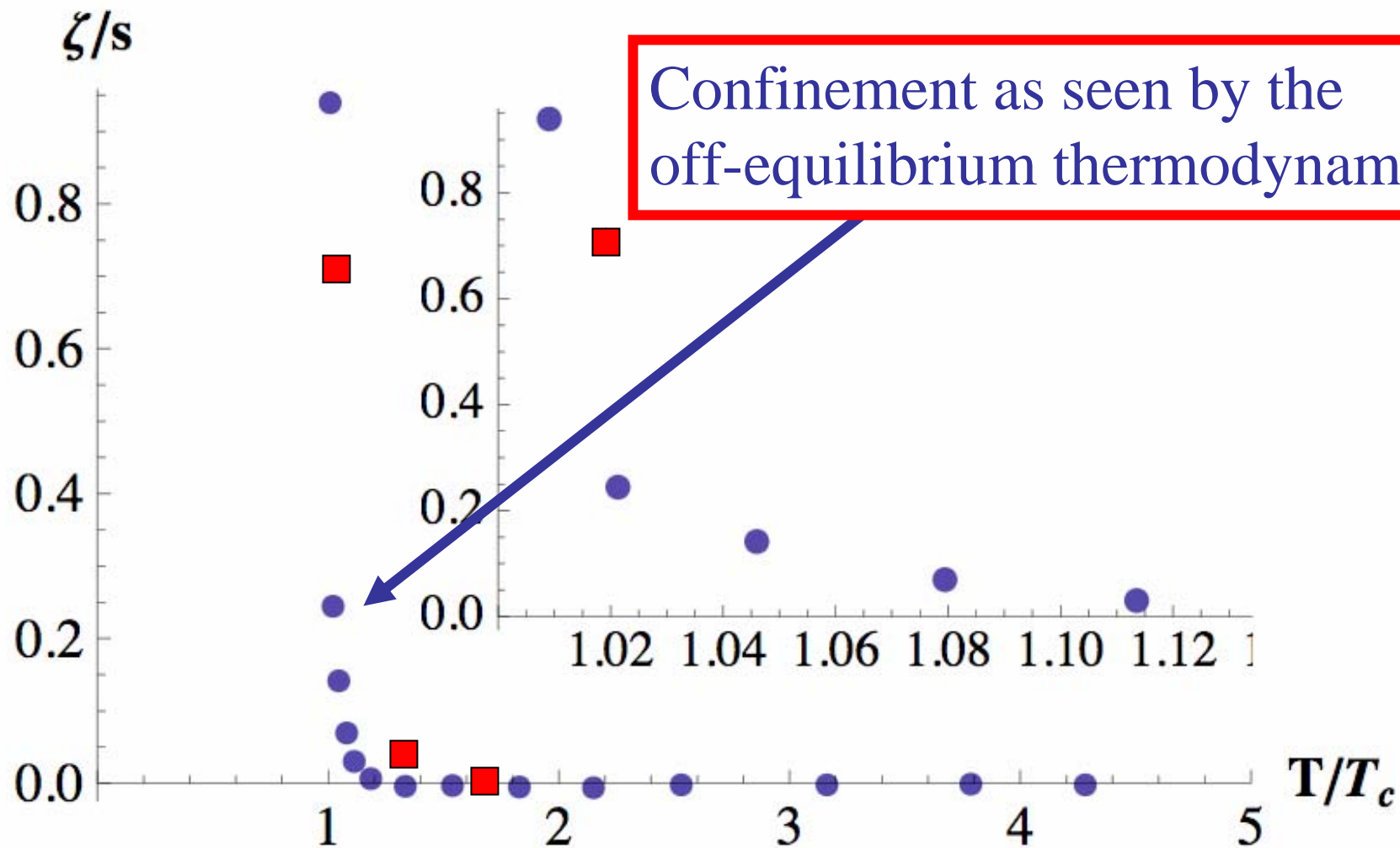
# Bulk viscosity and the mechanism of hadronization



**Growth of entropy**  $\longleftrightarrow$  **event horizon?**

● Kharzeev-Tuchin

■ Meyer



# Summary

1. Deep connections are likely to exist between gauge theories and gravity
2. Bulk viscosity grows dramatically (3 orders!) close to the critical temperature (most likely, a peak at  $T_c$ ): by far, the dominant viscous effect at this temperature
3. This suggests a new scenario for statistical hadronization; a link between the thermal hadron yields and an event horizon for colored particles?

Need to devise the methods of experimental study

...and finally,

# **THANK YOU, the Organizers!**

**Bikash Sinha      Helmut Satz**

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B. Mohanty

J. Alam

P. Roy

S. Chattopadhyay

T. Nayak

T. Samanta